Physics-informed Gaussian process model for Euler-Bernoulli beam elements

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Abstract

A physics-informed machine learning model, in the form of a multi-output Gaussian process, is formulated using the Euler-Bernoulli beam equation. Given appropriate datasets, the model can be used to regress the analytical value of the structure's bending stiffness, interpolate responses, and make probabilistic inferences on latent physical quantities. The developed model is applied on a numerically simulated cantilever beam, where the regressed bending stiffness is evaluated and the influence measurement noise on the prediction quality is investigated. Further, the regressed probabilistic stiffness distribution is used in a structural health monitoring context, where the Mahalanobis distance is employed to reason about the possible location and extent of damage in the structural system. To validate the developed framework, an experiment is conducted and measured heterogeneous datasets are used to update the assumed analytical structural model.

Keywords: Gaussian process; physics-informed; machine learning; stiffness regression; structural health monitoring, model updating.

1 Introduction

Machine learning has been extensively applied in structural engineering, especially in the structural health monitoring (SHM) field, as the availability of data collected from sensors increases [1]. The heterogeneity of the collected datasets along with the knowledge of physical relations between them, usually represented as partial differential equations (PDEs), have motivated the recent use of physics-informed machine learning models to extract meaningful information from measured data. In these algorithms, the governing PDE is built into the machine learning model, effectively integrating measurements and mathematical models [2]. Gaussian processes (GP) [3] have been extensively used for such a task, as they offer a non-parametric probabilistic view on the modelling scheme, which is usually framed in a Bayesian manner [4, 5]. Particular applications of the physics-informed GPs have been observed as an analytical model of the latent curvature in a sleeper beam and the inverse problem of identifying the Reynolds number of a CFD simulation [6, 7]. In this paper, a physics-informed Gaussian process model for the Euler-Bernoulli beam formulation is developed to simultaneously infer physical quantities of interest, whilst considering the problem of identifying the correct structural bending stiffness. The model is defined in section 2, along with the optimization strategies, while section 3 is used to demonstrate the inference capabilities and tolerances to measurement noise in a numerical and controlled manner. Section 4 utilizes experimental measurements to update an analytical model of a steel beam.

2 Physics-informed GP model of an Euler-Bernoulli beam

2.1 Model definition

Consider the general linear form of the Euler-Bernoulli beam equation: