

## A numerical algorithm to predict the behaviour of cracked r.c. members

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## Summary

The classical approach considering perfect bond between steel rebars and concrete is often unsuitable in refined analysis of reinforced concrete elements, as it leads to significant errors in the estimate of the actual stiffness. For this reason more refined procedures have to be considered. In the paper a numerical algorithm is illustrated, able to predict both the evolution of the crack pattern and the crack widths during the whole loading process, based on a refined local bond-slip law, derived from ad hoc tests.

**Keywords:** bond-slip, reinforced concrete, non linear behaviour, numerical analysis, crack pattern, crack opening.

## 1. Introduction

The actual non linear behaviour of reinforced concrete elements under static or dynamic loads is strongly dependent on the effective force-displacement or bending moment-rotation relationships for cracked members, so that the appropriate estimate of these diagrams, as well as of crack patterns and crack widths, represents a key aspect in refined structural analysis [1], [2], [3], [4], [5].

Experimental studies, [6], [7], [8] show that the classical approach base on the assumption of full bond between steel rebars and concrete is often unsuitable in refined analysis, leading to remarkable errors in the estimate of the actual stiffness of the elements. Concrete cracking and steel to concrete slip, which are usually ignored in the classical approach, play a significant role in the prediction of the structural deformations.

In the paper it is illustrated a numerical algorithm, which allows to predict the evolution of the crack pattern as well as the crack opening and relative steel to concrete slip in a reinforced concrete tie. In the proposed procedure, reference is made to a local bond-slip law which has been derived by the Authors trough ad hoc tests on r.c. ties [8]. Numerical examples are illustrated showing the outcomes of the application of the algorithm, in comparison with the prediction of EN1992-1-1 [9] and Model Code 90 [10].

## 2. Theoretical modelling of bond-slip in r.c. elements

Bond-slip analytical model, currently adopted in literature, is theoretically derived considering equilibrium and compatibility conditions of a r.c. infinitesimal cylindrical tie of length  $dx$ , made up by one single rebar and the surrounding portion of concrete in tension,  $A_c$ , fig. 1.

The equation governing the global equilibrium of the tie is