

# Composite Beam: Non-Linear Analytical Exact Fully-Developed Model

**Paolo FORABOSCHI**  
Professor of Structural  
Engineering, University IUAV  
of Venice, VE, Italy  
*paofor@iuav.it*

Paolo Foraboschi received his civil engineering degree from the Bologna University and his PhD from the Florence University (Italy). Paolo Foraboschi has been professor at the University IUAV of Venice since 1998, where he had been researcher. His main areas of research are related to seismic engineering, masonry, advanced and innovative materials and techniques, structural glass and glass structures, bridges.

## Summary

This two-page short version of the paper presents the application of the model to two steel-concrete composite beams, while the model is synthesized in the full version of the paper.

**Keywords:** Bi-layered; Elasto-Plastic interlayer; Inelastic interface; Layer beam; Nonlinear analytical model; Shear connection; Shear flow; Slip; Steel-Concrete; Strain-Softening interlayer; Stud.

## 1. Introduction

The research consists of a fully-developed nonlinear analytical (exact) model for analyzing composite beams under transverse bending load. The mathematical model reproduces the elements responsible for the relative slip between the layers (shear connectors and interface) with an elasto-plastic strain-softening interlayer. Further than the slip, the model predicts stresses due to a given load and ultimate load for debonding, of bi-layered composite beam. The details on the mathematical development are synthesized. This paper advances the state of the art, since the last development available in literature is an analytical (non-exact) linear model. A number of parametric studies were conducted in the research to evaluate the influence of various geometrical and material parameters, which main results are presented together with the interpretation, e.g., the dependence of load-carrying capacity, stresses, and deflection, on local nonlinear load-slip relationship. The research proves as well that the shear connection lower and upper bounds (respectively, totally flexible and infinite rigid shear connectors) do not imply any lower and upper bound for the response.

Behavior of shear connection falls between two intuitive bounds. The lower bound occurs for zero slip-stiffness shear connection. At this bound, layers slide on each other without receiving any resistance from interface (*freely sliding condition*). Thus, longitudinal shear forces (or stresses, if joined continuously) and composite action are nil. The upper bound occurs for infinite slip-stiffness shear connection. At this bound, layers do not slide on each other (*monolithic condition*). Thus, longitudinal shear force (or  $\sigma$ ) can be derived from Jourawsky's formula, and slip between layers is nil.

Shear connections that ensure negligible slip are expensive and time-consuming and do not provide substantial advantage; so they are hardly realized in practice. Conversely, practical applications prefer flexible (semirigid) shear connections. Since flexible shear connection implies non-negligible slips, only a fraction of the Jourawsky's longitudinal shear force is exchanged through the interface.

## 2. Mathematical model

To govern the slip with a differential equation rather than with a set of algebraic equations, the model smears the shear flow through the whole interface. The schematization that represents the smeared shear flow is a beam composed of two distinct layers — upper,  $A$  and lower,  $B$  — and a connecting continuous interlayer along the layers interface (Fig. 1). The continuous interlayer exchanges shear force per unit area of interface,  $\tau_i$ , with the two adjacent layers (continuous shear flow), whereas the actual shear connectors exchange concentrated longitudinal shear forces  $T$  (Figs. 2,3). Interlayer behavior is governed by the shear stress  $\tau_i$  versus the slip  $\zeta$  relationship (Fig. 4).